CIRCLES

SSLC - MATHEMATICS CHAPTER 10 CIRCLES ENGLISH VERSION

English version

Yakub KOYYUR , GHS Nada,Belthangady Taluk, D.K., 574214 Ph:9008983286

Chapter -10
Circles

Main point to be Remember

- Equal chords are equidistant from the centre.
- Angles in the same segment are equal.
- Angles in the major segment are acute angles.
- Angles in the minor segment are obtuse angles.
- Angles in a semi-circle are right angles.

CONGRUENT CIRCLES

Circles having same radii but different centers are called congruent circles

CONCENTRIC CIRCLE

Circles having the same center but different radii are called concentric circles.

SECANT

A straight line which intersects a circle at two distinct points is called a Secant

TANGENT

A straight line which touches the circle at only one point is called a tangent..

POINT OF CONTACT

The point where a tangent touches the circle is called the point of contact

CHORD

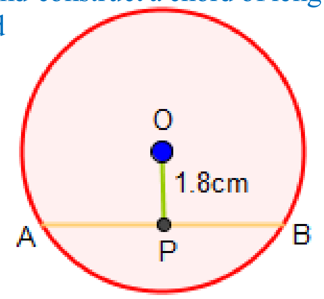
The line segment joining any two points on the circle..

Characteristics of Tangents

- In any circle, the radius drawn at the point of contact is perpendicular to the tangent.
- The perpendicular to the radius at its non-centre end is the tangent to the circle.
- Observe that, in a circle angle between the radii and angle between the tangents drawn at their non-centre ends are supplementary.
- The perpendicular to the tangent at the point of contact passes through the centre of the circle.
- Tangents drawn at the ends of a diameter are parallel to each other
- Only two tangents can be drawn from an external point to a circle
- Only one tangent can be drawn to a circle at any point on it.
- The tangents drawn from an external point to a circle are equal.
- Two circles having only one common point of contact are called touching circles.
- If two circles touch each other externally, the distance between their centres is equal to the sum of their radii [$d = R + r$]
- If two circles touch each other internally, the distance between their centres is equal to the difference of their radii [$d = R - r$]
- If two circles touch each other, their centres and the point of contact are collinear.
- If both the circles lie on the same side of a common tangent, then the common tangent is called a direct common tangent (DCT)
- If both the circles lie on either side of a common tangent, then the common tangent is called a transverse common tangent (TCT).
- Length of the tangent drawn from an external point $t = \sqrt{d^2 - r^2}$
- DCT $t = \sqrt{d^2 - (R - r)^2}$
- TCT $t = \sqrt{d^2 - (R + r)^2}$

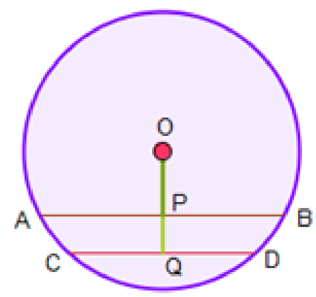
Exercise 10.1

1. Draw a circle of radius 3.5 cm and construct a chord of length 6 cm in it. Measure the distance between the centre and the chord



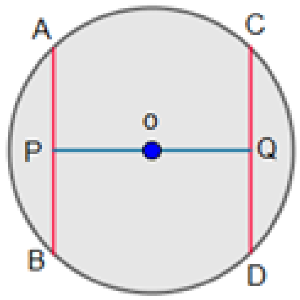
Distance between the centre and the chord = 1.8cm

2. Construct two chords of length 6 cm and 8 cm on the same side of the centre of a circle of radius 4.5cm. Measure the distance between the centre and the chords .



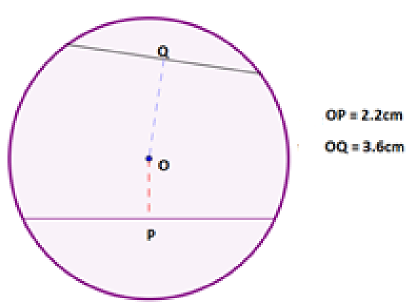
Chord OP = 2.1cm (2.06) and OQ = 3.3cm

3. Construct two chords of length 6.5cm each on either side of the centre of a circle of radius 4.5 cm. Measure the distance between the centre and the chords.



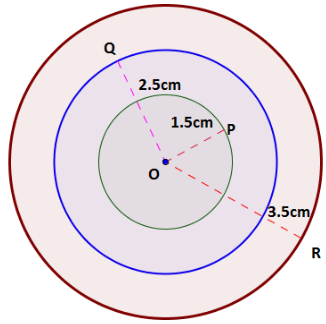
Distance between the chord and the centre is 3.1cm

4. Construct two chords of length 9cm and7 cm on either side of the centre of a circle of radius 5 cm. Measure the distance between the centreand the chord.

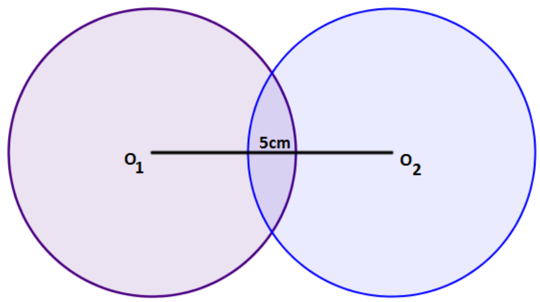


Exercise10.2

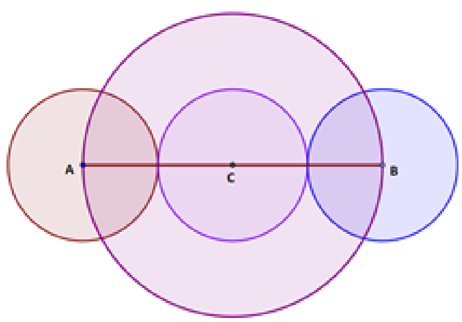
1. Draw three concentric circles of radii 1.5 cm, 2.5cm and 3.5cm with O as centre.



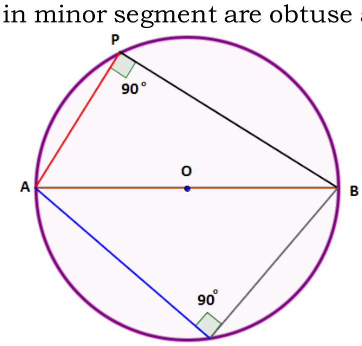
2. With O_1 and O_2 as centres draw two circles of same radii 3 cm and with the distance between the two centres equal to 5 cm



3. Draw a line segment AB = 8 cm and mark its mid point as C. With 2 cm as radius draw three circles having A, B and C as centres. With C as centre and 4 cm radius draw another circle. Identify and name the concentric circles and congruent circlesAB = 8cm

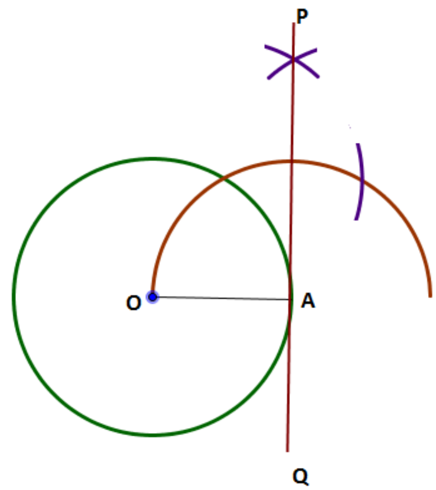


4. Draw a circle of radius 4 cm and construct a chord of 6 cm length in it. Draw two angles in major segment and two angles in minor segment. Verify that angles in major segment are acute angles and angles in minor segment are obtuse angles by measuring them

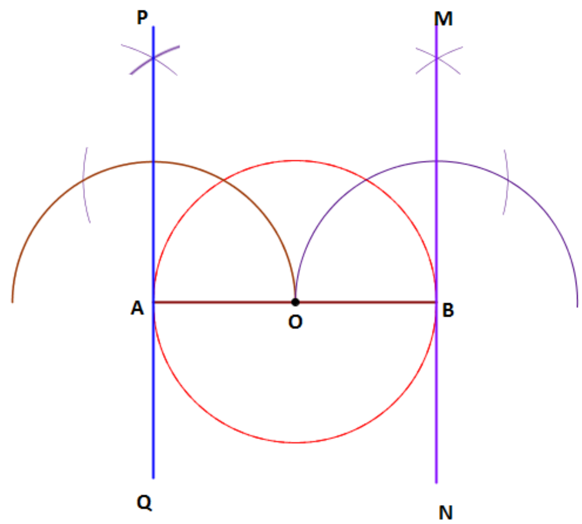


Exercise10.3

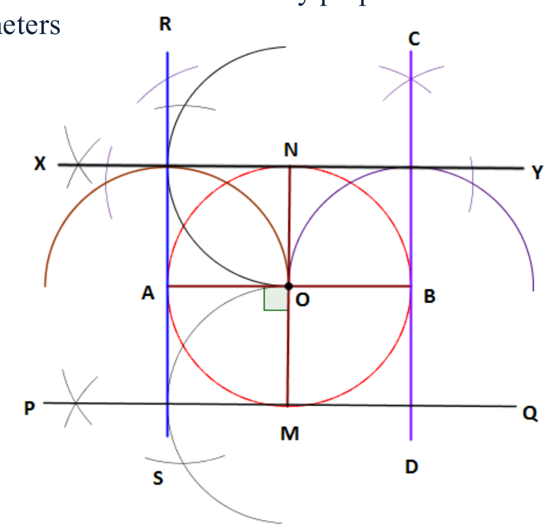
1. Draw a circle of radius 4 cm and construct a tangent at any point on the circle 4cm .



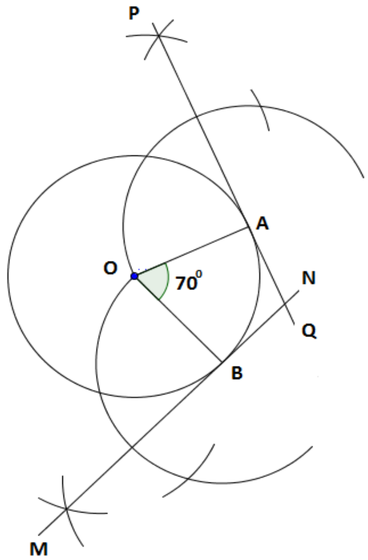
2. Draw a circle of diameter 7 cm and construct tangents at the ends of a diameter



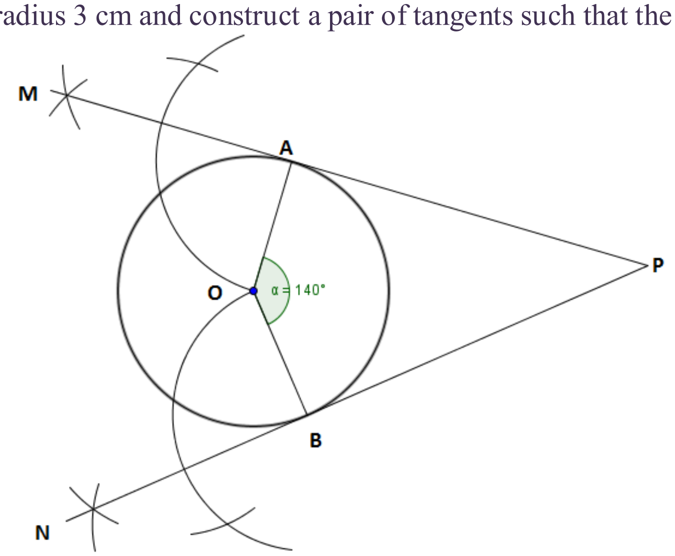
3. In a circle of radius 3.5cm draw two mutually perpendicular diameters. Construct tangents at the ends of the diameters



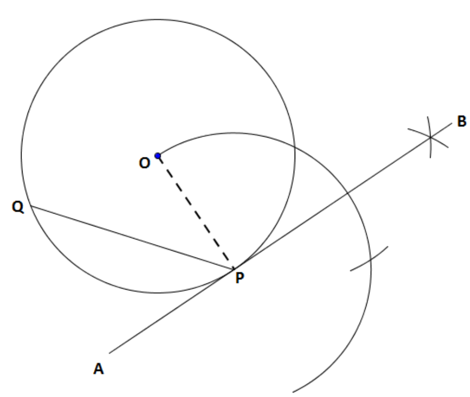
4. In a circle of radius 4.5 cm draw two radii such that the angle between them is 70°. Construct tangents at the non-centre ends of the radii.



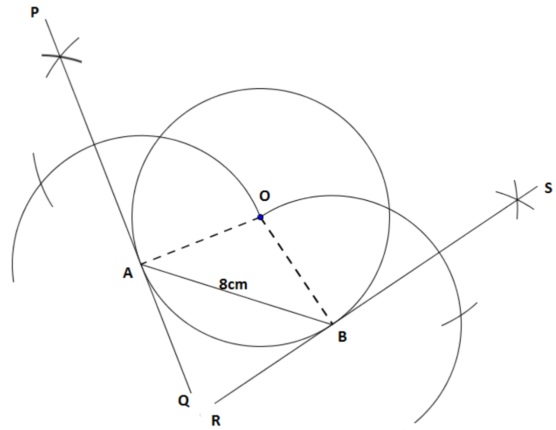
5. Draw a circle of radius 3 cm and construct a pair of tangents such that the angle between them is 40°.



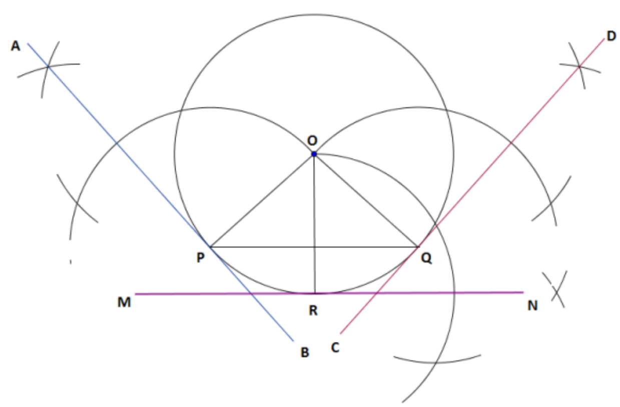
6. Draw a circle of radius 4.5 cm and a chord PQ of length 7cm in it. Construct a tangent at P.



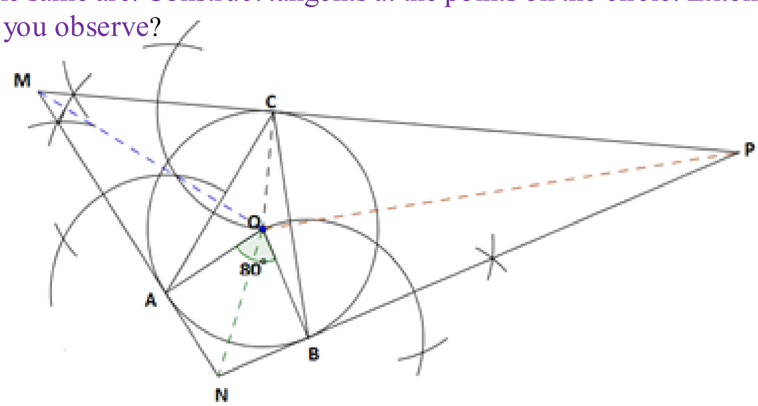
7.In a circle of radius 5 cm draw a chord of length 8 cm. Construct tangents at the ends of the chord.



8. Draw a circle of radius 4cm and construct chord of length 6 cm in it. Draw a perpendicular radius to the chord from the centre. Construct tangents at the ends of the chord and the radius.

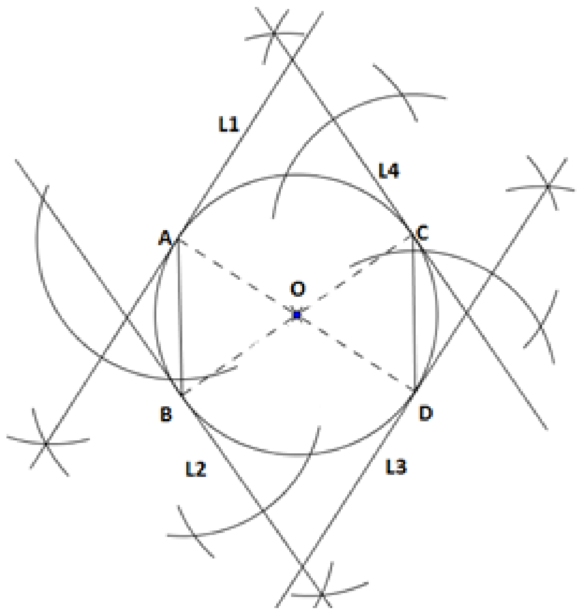


9. Draw a circle of radius 3.5 cm and construct a central angle of measure 80° and an inscribed angle subtended by the same arc. Construct tangents at the points on the circle. Extend tangents to intersect. What do you observe?



Extend tangents to intersect we get a tringle

10. In a circle of radius 4.5cm draw two equal chords of length 5cm on either sides of the centre. Draw tangents at the end points of the chords .



∴

ILLUSTRATED EXAMPLES

Example 1 : In the figure, a circle is inscribed in a quadrilateral ABCD in which ∠B =90°. If AD = 23cm, AB = 29cm and DS = 5cm find the radius of the circle.

Sol: In the figure. AB, BC, CD and DA are the tangents drawn to the circle at Q, P, S and R respectively.

DS = DR (tangents drawn from an external point D to the circle.)

but DS = 5cm (given)

∴ DR = 5 cm

In the fibure AD = 23cm (given)

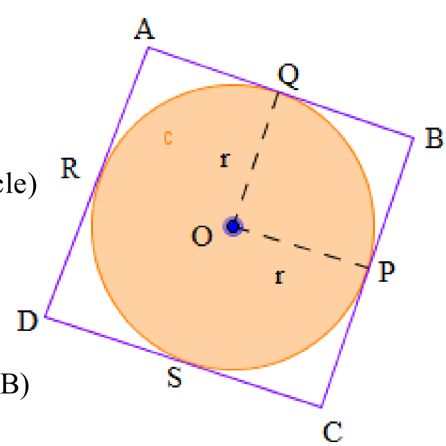
∴ AR = AD – DR = 23 – 5 = 18 cm

but AR = AQ

(tangents drawn from an external point A to the circle)

∴ AQ = 18 cm.

If AQ = 18 cm then (given AB = 29 cm)



BQ = AB – AQ = 29 – 18 = 11 cm

In a quadrilateral BQOP,

BQ = BP (tangents drawn from an external point B)

OQ = OP (radii of the same circle)

∠QBP = ∠QOP= 90° (given)

∠OQB = ∠OPB = 90° (angle between the radius and the tangent at the point of contact.)

∴ BQOP is a square.

$\therefore$ radius of the circle, OQ = 11 cm

Example: In the fig. AP and BP are the tangents drawn from an external point P. Prove that $\angle AOB$ and $\angle APB$ are supplementary.

Sol: Data : O is the centre of the circle. P is an external point. PA and PB are the tangents drawn from an external point P.

To prove : $\angle AOB + \angle APB = 180°$

Proof : In quadrilateral AOBP,

$\angle AOB + \angle OBP + \angle BPA + \angle PAO = 360°$

but $\angle OBP = 90^0$ and $\angle OAP = 90°$

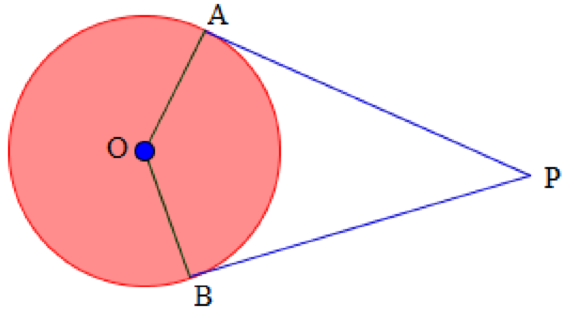
$\therefore AOB + 90^0 + \angle BPA + 90^0 = 360°$

$\angle AOB + \angle BPA + 180^0 = 360°$

$\angle$ $\angle AOB + \angle BPA = 360° - 180°$

$\angle AOB + \angle BPA = 180^0$

$\therefore$ $\angle AOB$ and $\angle BPA$ are supplementary.



Example 3 : In the figure, PQ and PR are the tangents to a circle with centre O. If $\angle P = \frac{4}{5}\angle O$ find $\angle O$ and $\angle P$.

Sol: In the figure $\angle QPR + \angle QOR = 180°$

[$\because$ Opposite angles are supplementary]

But, $\angle QPR = \frac{4}{5}\angle QOR$

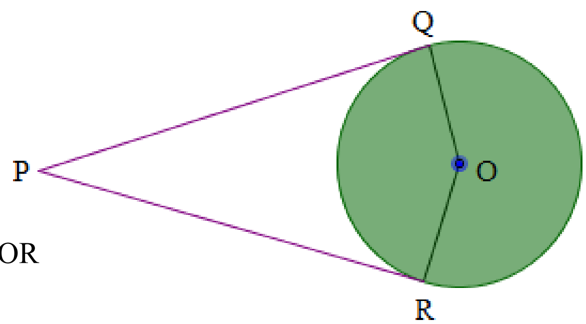
$\therefore \frac{4}{5}\angle QOR + \angle QOR = 180^0$

$\Rightarrow \angle QOR = \frac{5x\ 1800}{9} = 100^0$

If $\angle QOR = 100°$ then $\angle QPR = 180° - \angle QOR$

$= 180° - 100° = 80°$

$\therefore \angle QPR = 80^0$



Example 4 : In the figure O is the centre of the circle. The tangents at B and D intersect each other at point P. If AB is parallel to CD and $\angle ABC = 55°$ Find (i) $\angle BOD$ (ii) $\angle BPD$

Sol: In the figure AB ∥ CD (given)

$\therefore$ $\angle ABC = \angle BCD$ ($\because$ alternate angles)

But, $\angle ABC = 55°$ ($\because$ given)

$\angle BCD = 55°$

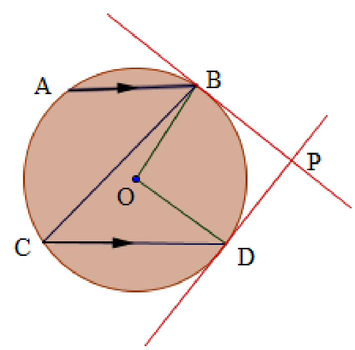
In the figure $\angle BOD = 2\ \angle BCD$

(central angle is twice the inscribed angle.)

$\therefore$ $\angle BOD = 2 \times 55° = 110°$.

But, $\angle BOD + \angle BPD = 180°$

$\therefore \angle BPD = 180° - \angle BOD = 180° - 110° = 70°$



Example 5 : In the figure, show that perimeter of $\Delta ABC = 2$ (AP + BQ + CR).

Sol: Perimeter of ΔABC = AB + BC + AC

= AP + PB + BQ + QC + CR + RA

But, AP = AR (tangents drawn from A to the circle)

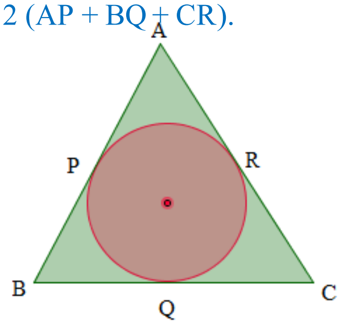
PB = BQ (tangents drawn from B to the circle)

CQ = CR (tangents drawn from C to the circle)

$\therefore$Perimeter of ΔABC = AP + BQ + BQ + CR + CR + AP

= 2AP + 2BQ + 2CR

= 2(AP + BQ + CR)

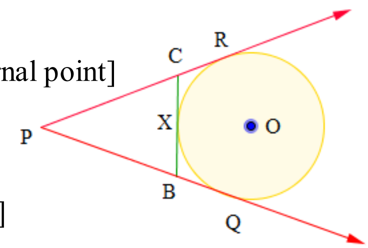


Exercise 10.4

A. Numerical problems based on tangent properties

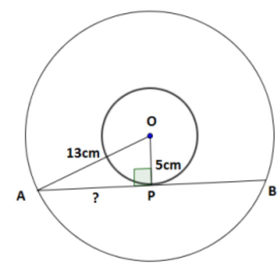
1. In the figure PQ, PR and BC are the tangents to the circle BC touches the circle at X. If PQ = 7cm, find the perimeter of ΔPBC

The perimeter of ΔPBC = PC + PB + BC
= PC + PB + BX + CX
But CX =CR and BX = BQ [∵tangents drawn from an external point]
∴ PC + PB + BQ + CR
= (PC + CR) + (PB + BQ)
= PR + PQ
= 7 + 7 [∵ PR=PQ, tangents drawn from an external point]
= 14cm



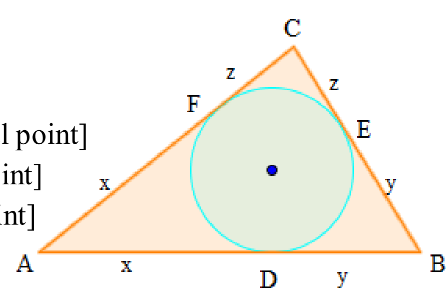
2. Two concentric circles of radii 13cm and 5cm are drawn Find the length of the chord of the outer circle which touches the inner circle.

O' is the centre, AB is the tangent drawn to a inner circle through the point P
AP = PB [∵ The perpendicular drawn from the point of contact devides the chord equally]
In ΔOAP , ∠OPA = 90°
∴ $AP^2 + OP^2 = OA^2$ [∵Pythagoras Theroem]
⇒ $AP^2 + 5^2 = 13^2$
⇒ $AP^2 + 25 = 169$
⇒ $AP^2 = 169-25$
⇒ $AP^2 = 144$
⇒ AP = 14cm
∴ Chord AB = AP + PB = 14 + 14 = 28cm



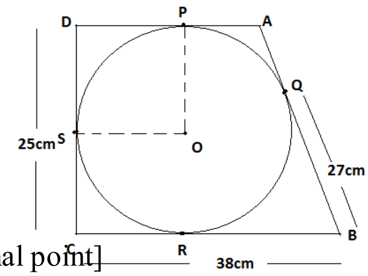
3. In the given ΔABC, AB = 12cm, BC = 8 cm and AC = 10cm. Find AF, BD and CE

In ΔABC , AB = 12cm, BC = 8cm and AC = 10cm
AB + BC + CA = 12 + 8 + 10
⇒ AD + BD + BE + CE + AF + CF = 30
But AF = AD = x [∵tangents drawn from an external point]
BE = BD = y [∵tangents drawn from an external point]
CE = CF = z [∵tangents drawn from an external point]
∴ x + y + y + z + z + x = 30
⇒ 2x + 2y + 2z = 30
⇒ x + y + z = 15 ------(1)
AB = x+y = 12cm, BC = y+z=8cm, AC = x+z = 10cm
(1)⇒ 12 + z = 15 [∵ x + y = 12cm]
∴ z = 15 – 12
∴ z = 3cm
(1)⇒ x + 8 = 15 [∵ y + z = 8cm]
∴ x = 15 – 8
∴ x= 7cm ,BD = y = 5cm,
(1)⇒ y + 10 = 15 [∵ x + z = 10cm]
∴ y = 15 – 10
∴ y= 5cm
∴ AF = x = 10cm,
BD = y = 5cm , CE = z = 3cm



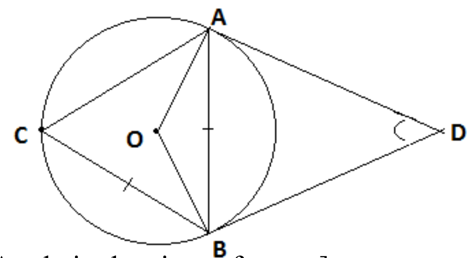
4. In the given quadrilateral ABCD,BC = 38cm, QB = 27cm, DC = 25cm and AD⊥DC find the radius of the circle .

In the figure OPDS ,
DS = DP [∵tangents drawn from an external point]
OP = OS [∵ Radius of a circle]
∠D = 90^0 [∵AD⊥DC]
∴ OPDS is a square.



In the figure,
BQ = BR = 27 cm [∵tangents drawn from an external point]
CR = 38 – 27 = 11cm = CS [∵tangents drawn from an external point]
DS = 25 -11 = 14cm = DP [∵tangents drawn from an external point]
∴ Radius of the circle = OP = OS = 14cm [∵OPDS is a square.]

5. In the given figure AB = BC, ∠ABC = 68^0 DA and DB are the tangents to the circle with centre are the tangents to the circle with centreO calculate the measure of Calculate the measure of
(i) ∠ACB (ii) ∠AOB ಮತ್ತು (iii) ∠ADB

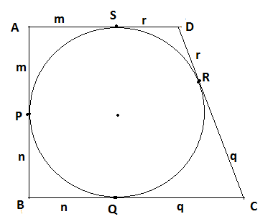


In the figre, AB = BC, ∠ABC = 68^0
(i).∠ACB =∠BAC = $180^0 – 68^0$
$= \frac{180^0 - 68^0}{2} = \frac{112}{2} = 56^0$ [∵AB = BC]
(ii). ∠AOB= 2x∠ACB = 2x56 = 112^0
[∵The angle forming from an arc at the centre =2xAngle in the circumference]
(iii). ∠ADB = 180^0-∠AOB =$180^0 – 112^0$ = 68^0
[∵Angle between tangents + angle in the centre =180]

B. Raiders Based on tangent properties

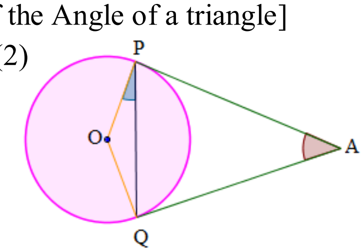
1. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB+CD=AD+BC.

AP = AS = m [∵tangents drawn from an external point]
BP = BQ = n [∵tangents drawn from an external point]
CQ = CR = q [∵tangents drawn from an external point]
DR = DS = r [∵tangents drawn from an external point]
AB + CD = AP + BP + CR + DR = m + n + q + r --------(1)
AD + BC = AS + DS + BQ + CQ
= m + r + n + q =m + n + q+ r---------(2)
From (1) and (2)
AB + CD = AD + BC

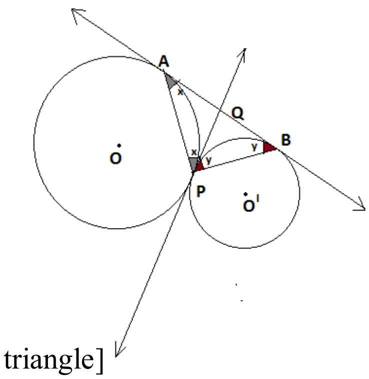


2. Tangents AP and AQ are drawn to circle with centre O, from an external point A. Prove that ∠PAQ = 2∠OPQ .

∠POQ + ∠PAQ = 180^0 **[**∵Angle between tangents + angle in the centre =180**]** ---------(1)
ΔPOQ ನಲ್ಲಿ ∠OPQ = ∠OQP [∵OP=OQ,Radius of the circle]
In ΔPOQ, ∠POQ + ∠OPQ + ∠OQP = 180^0 [∵Sum of the Angle of a triangle]
∴ ∠POQ + 2∠OPQ = 180^0 [∵∠OPQ = ∠OQP] -------(2)
(1) ಮತ್ತು (2) ರಿಂದ,
∠POQ + ∠PAQ = ∠POQ + 2∠OPQ
∠PAQ = 2∠OPQ

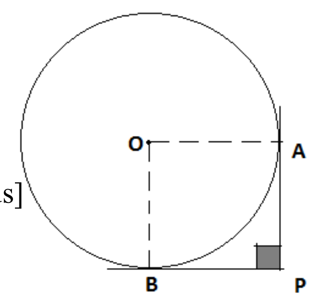


3. In the figure two circles touch each other externally at P. AB is a direct common tangent to these circles. Prove that,
(a) tangent at P bisects AB at Q .
(b) $\angle APB = 90^0$.



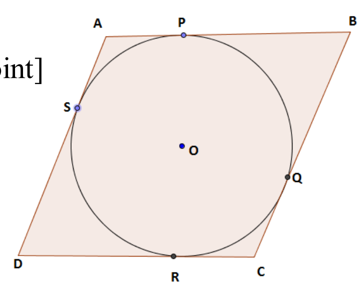
(a) In the figure,
QA = QP [∵tangents drawn from an external point]---(1)
QB = QP [∵tangents drawn from an external point]---(2)
(1) and (2)
QA = QB
∴ P bisects AB at Q.
(b) .In ΔAPB,
$\angle QAP = \angle APQ = x$ [∵ QA = QP]
$\angle QPB = \angle BPQ = y$ [∵ QB = QP]
∴ ΔAPB ಯಲ್ಲಿ $x + x + y + y = 180^0$ [∵Sum of the Angle of a triangle]
$\Rightarrow 2x + 2y = 180^0$
$\Rightarrow x + y = 90^0$
$\Rightarrow \angle APB = 90^0$

4. A pair of perpendicular tangents are drawn to a circle from an external point. Prove that length of each tangent is equal to the radius of the circle



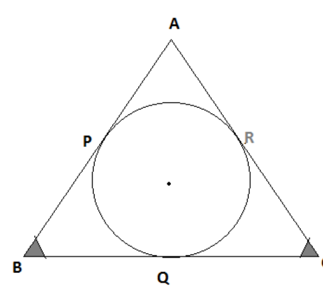
In the figure,
PA = PB [∵ tangents drawn from an external point]
OA = OB [∵ Radius of the circle]
$\angle APB = 90^0$ [∵ Given]
$\angle OAP = \angle OBP = 90^0$ [∵ The angle between tangent and the radius]
∴ $\angle AOB = 90^0$ [∵ The sum of angles of a quadrilateral = 360^0]
∴ OABP is a square
∴ Tangents (PA and PB) = Radius of the circle (OA and OB)

5. If the sides of a parallelogram touch a circle. Prove that the parallelogram is a rhombus.
Let ABCD be a parallelogram.



∴ AB∥CD, AB = CD [∵ Given]
AD∥BC, AD = BC [∵ Given]
AP = AS, PB = BQ [∵ tangents drawn from an external point]
DS = DR, QC = RC
⇒AB + CD = AP +PB + CR + DR
⇒AB + CD = AS + BQ + QC + DS
⇒AB + CD = (AS + DS) +(BQ + CQ)
⇒AB + CD = AD + BC
⇒2AB = 2AD [∵ ದತ್ತ AB = CD; AD = BC]
⇒AB = AD
⇒AB = AD = CD = BC
⇒ABCD is a rhombus [∵ Side of the parallelogram are equal.

5. In the figure, if AB = AC prove that BQ = QC .

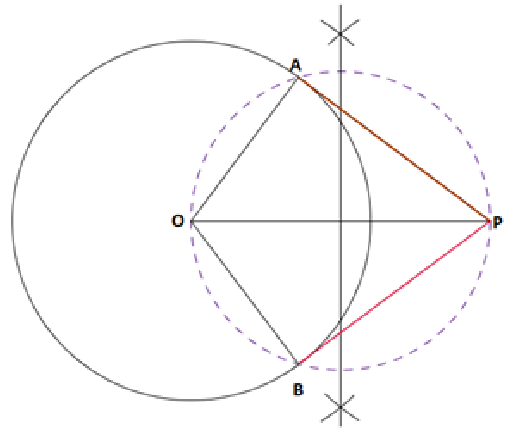


AP = AR -----(1) [∵tangents drawn from an external point]
and AB = AC ---------(2) [∵Given]
(1)– (2)

∴ AB – AP = AC - AR
∴ BP = CR
But, BQ = BP and CQ = CR [∵tangents drawn from an external point]
∴ BQ = CQ

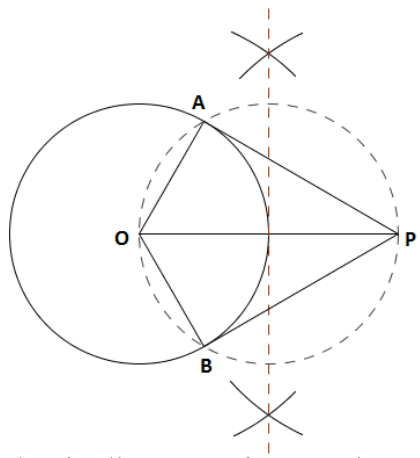
Exercise 10.5

1. Draw a circle of radius 6 cm and construct tangents to it from an external point 10 cm away from the centre. Measure and verify the length of the tangents

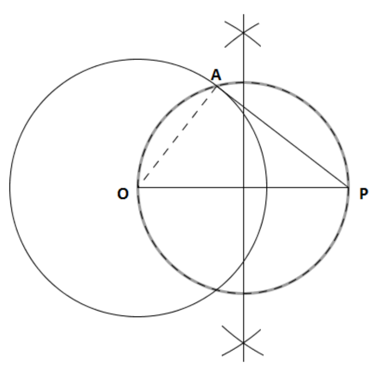


Length of the tangent t = $\sqrt{d^2 - r^2}$
Length of the tangent t = $\sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8cm$

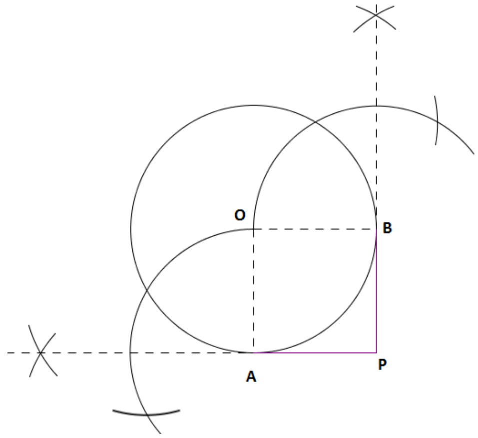
2. Construct a pair of tangents to a circle of radius 3.5cm from a point 3.5cm away from the circle



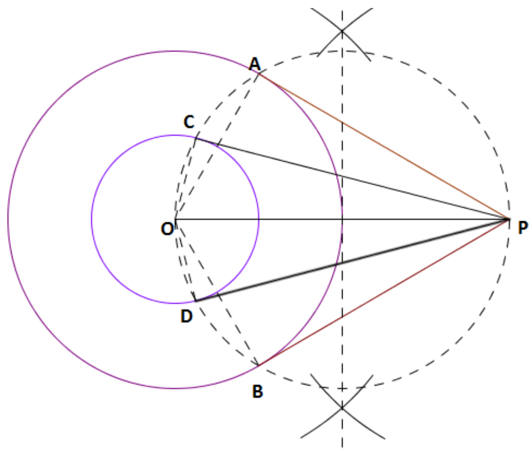
3. Construct a tangent to a circle of radius 5.5cm from a point 3.5 cm away from it 5cm



4. Draw a pair of perpendicular tangents of length 5cm to a circle



5. Construct tangents to two concentric circles of radii 2cm and 4cm from a point 8cm away from the centre

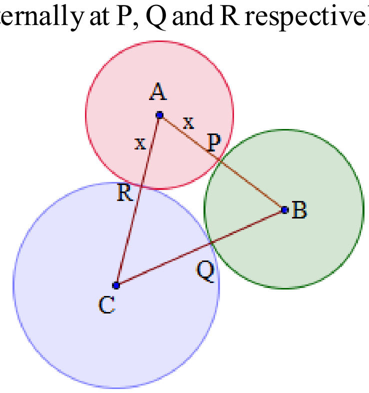


ILLUSTRATIVE EXAMPLES

Example 1.Three circles touch each other externally. Find the radii of the circles if the sides of the triangle obtianed by joining the centres are 10cm,14cm and 16cm respectively.

Sol. Circles with centres A, B and C touch each other externally at P, Q and R respectively

As shown in the figure,



Let AP = x

$\therefore PB = BQ = 10 - x$

$RC = CQ = 14 - x$

But, $CQ + BQ = 16$

$14 - x + 10 - x = 16$

$24 - 2x = 16$

$24 - 16 = 2x$

$2x = 8$

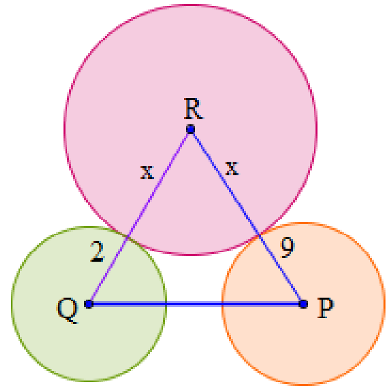
$x = 4$

$\therefore$ AR = AP = 4cm radius of the circle with centre A

BQ = 10 – 4 = 6cm radius of the circle with centre B
CR = 14 – 4 = 10cm radius of the circle with centre C

Example 2. In the figure P and Q are the centres of the circles with radii 9cm and 2cm respectively. If ∠PRQ 90° and PQ 17cm find the radius of the circle with centre R

Let the radius of the circle with centre R = x units.



∴In Δ PQR
PQ = 17cm
PR = (x + 9) cm
QR = (x + 2)cm and ∠PRQ = 90°
R = (x + 2) cm and PRQ = 90°

$\therefore PQ^2 = PR^2 + QR^2$ (Pythagoras theorem)

$17^2 = (x+9)^2 + (x+2)^2$

$289 = x^2 + 81 + 18x + x^2 + 4 + 4x$

$2x^2 + 22x + 85 - 289 = 0$

$2x^2 + 22x - 204 = 0 \quad \div \text{ by } 2$

$x^2 + 11x - 102 = 0 \Rightarrow (x+17)(x-6) = 0$

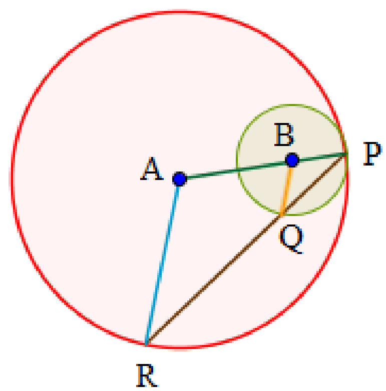
$\Rightarrow x + 17 = 0 \text{ or } x - 6 = 0$

$x = -17 \text{ or } x = 6$

∴ radius of the circle with centre R = 6cm

Example 3. In the figure circles with centres A and B touch each other internally. P is the point of contact. Prove that AR || BQ.

Sol: In the figure, Let ∠BPQ = x^0 .



In ΔPBQ ,
BP = BQ [∵Radii of the same circle]
∴ ∠BQP = ∠BPQ
[∵angles opposite to equal sides of an isosceles □le]
∴ ∠BQP = x^0 --------(1) [∵∠BPQ = x^0]
Similarly, In ΔPAR,
AP = AR [∵Radii of the same circle]
∠ARP = ∠APR
[∵angles opposite to equal sides of an isosceles □le]
∴ ∠ARP = x^0 --------(2) [∵∠APR = x^0]
From (1) and (2),
∠BQP = ∠ARP
But, ∠BQP and ∠ARP are corresponding angles
∴AR ∥ BQ

Exercise 10.6

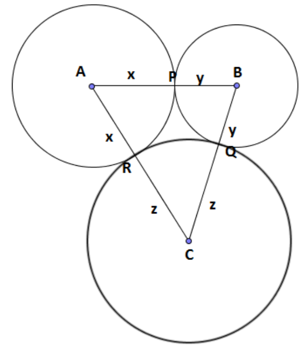
A. Numerical problems on touching circles.

1. Three circles touch each other externally. Find the radii of the circles if the sides of the triangle formed by joining the centres are 7cm, 8cm and 9cm respectively .

In the figure,
Let Radius of the circles be AP = x, BQ = y ಮತ್ತು CR = z .
AB = AP+BP = x+y = 7cm
BC = BQ+CQ= y+z = 8cm
AC = CR+AR= z+x = 9cm

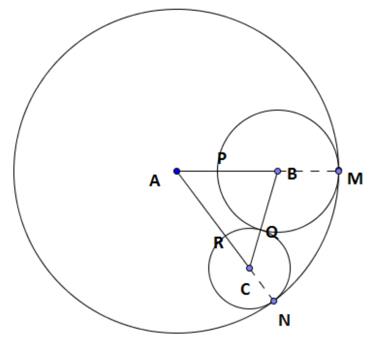
The perimeter of ΔABCಯ $\Rightarrow$ AB + BC + AC = 7 +8 + 9 = 24

$\Rightarrow$ AP + BP + BQ + CQ +CR + AR=24
$\Rightarrow$ x + y + y + z +z + x = 24
$\Rightarrow$ 2x + 2y + 2z = 24
$\Rightarrow$ x + y + z = 12
7 + z = 12 $\Rightarrow$ z = 12 -7 = 5cm [∵x + y =7]
x + 8 = 12 $\Rightarrow$ x = 12 -8 = 4cm [∵y + z =8]
y + 9 = 12 $\Rightarrow$ y = 12 -9 = 3cm [∵z + x =9]



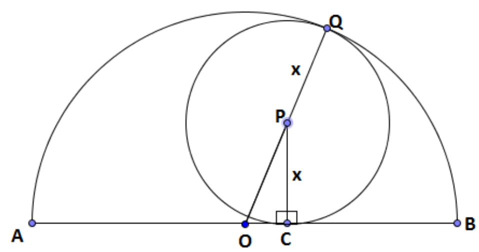
2. Three circles with centres A, B and C touch each other as shown in the figure. If the radii of these circles are 8 cm, 3 cm and 2 cm respectively, find the perimeter of ΔABC.

The perimeter of ΔABC.= AB + BC + AC
AB = AM – BM = 8 – 3 = 5cm
BC = BQ + CQ = 3 + 2 = 5cm
AC = AN – CN = 8 – 2 = 6cm
∴ The perimeter of ΔABC = AB + BC + AC
= 5 + 5 + 6 = 16cm



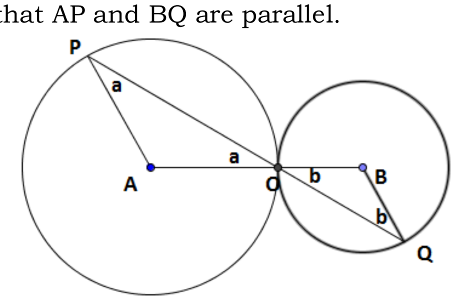
3. In the figure AB = 10 cm,AC = 6cm and the radius of the smaller circleis x cm.Find ನ x.

ΔOPC ಯಲ್ಲಿ, ∠PCO = 90^0
∴ $PC^2 = OP^2 - OC^2$
∴ $x^2 = (OQ – PQ)^2 - (AC – OA)^2$
[∵ OP = OQ – PQ, OC = AC – AO]
∴ $x^2 = (5 – x)^2 - (6 – 5)^2$ [∵ OQ = OA=5]
∴ $x^2 = 25 –10x + x^2 – 1$
∴ 10x = 24 $\Rightarrow$ x = 2.4cm

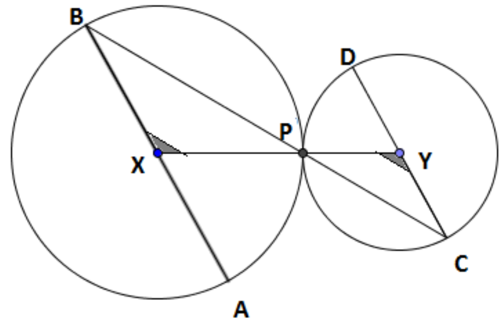


B. Raiders based on touching circles.

1. A straight line drawn through the point of contact of two circles with centres A and B intersect the circles at P and Q respectively. Show that AP and BQ are parallel.
∠AOP = BOQ [∵ Vertically opposite angles]
∠APO = ∠AOP [∵ AO=AP Radius of the circle]
∠BQO = ∠BOQ
$\Rightarrow$ ∠APO = ∠BQO
Thes are alternate angles,
∴ AP∥BQ

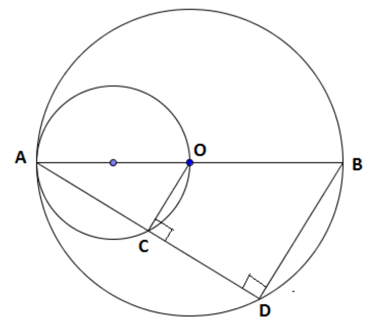


2. Two circles with centres X and Y touch each other externally at P. Two diameters AB and CD are drawn one in each circle parallel to other. Prove that B, P and C are collinear.
∠BXP = ∠PYC [∵Alternate angles AB∥CD]



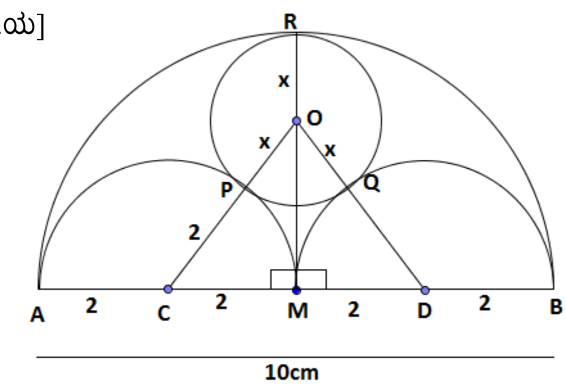
$\angle BPX = \angle PBX$ [∵XB=XP Radii]
∴ $\angle BPX + \angle PBX + \angle BXP = 180^0$
∴ $2\angle BPX + \angle BXP = 180^0$-------(1)
$\angle CPY = \angle PCY$ [∵YP=YC Radii]
∴ $\angle CPY + \angle PCY + \angle PYC = 180^0$
∴ $2\angle CPY + \angle PYC = 180^0$-------(2)
From(1) and (2) .
$2\angle BPX + \angle BXP = 2\angle CPY + \angle PYC$
⇒ $2\angle BPX = 2\angle CPY$
⇒ $\angle BPX = \angle CPY$
These are vertically opposite angles
∴ B,P,C are collinear.

3. In circle with centre O, diameter AB and a chord AD are drawn. Another circle is drawn with OA as diameter to cut AD at C. prove that BD = 2 OC.



$\angle ADB = 90^0$ [∵ Angle of semicircle]
$\angle ACO = 90^0$ [∵ Angle of semicircle]
In ΔADB and ΔAOC,
$\angle ADB = \angle ACO = 90^0$
$\angle A = \angle A$
∴ $\Delta ADB \sim \Delta AOC$
∴ By B.P.T. $\frac{BD}{OC} = \frac{AB}{AO}$
⇒ $\frac{BD}{OC} = \frac{2AO}{AO}$ [∵ AB=2AO]
⇒ $\frac{BD}{OC} = 2$
⇒ $BD = 2OC$

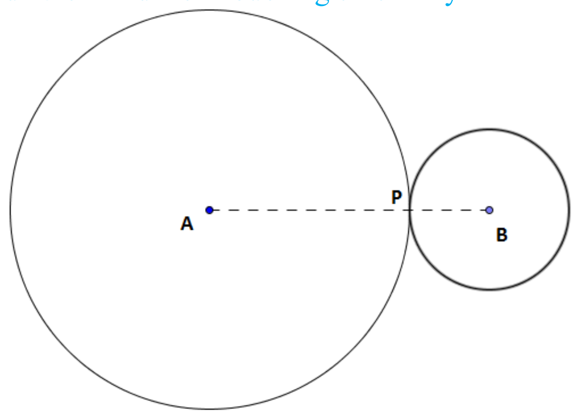
4. In the given figure AB = 8cm, M is the mid point of AB A circle with centre 'O' touches all three semicircles as shown. Prove that the radius of this circle is shown. Prove that the radius of this circle is $\frac{1}{6}AB$.



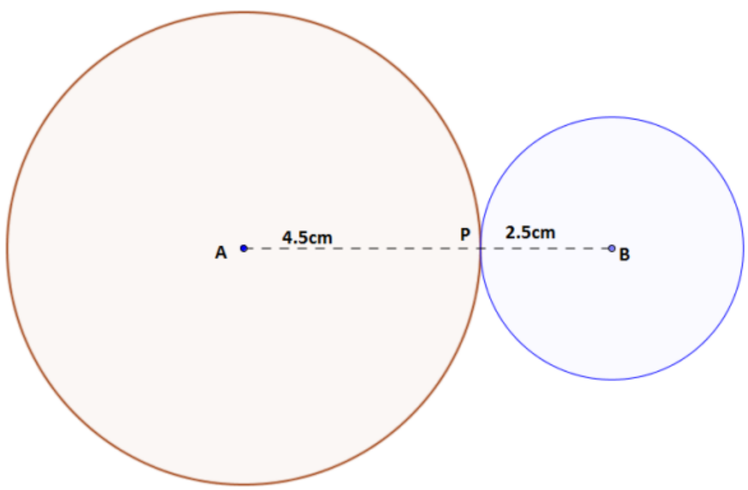
ΔOPC ಯಲ್ಲಿ, $\angle POC = 90^0$
∴ $OC^2 = OM^2 + MC^2$ [∵ ಪೈಥಾಗೊರಸ್ ಪ್ರಮೇಯ]
∴ $(CP+OP)^2 = (MR-OR)^2 + MC^2$
∴ $(2 + x)^2 = (4 - x)^2 + 2^2$
∴ $4 + 4x + x^2 = 16 – 8x + x^2 + 4$
∴ $4 + 4x = 16 – 8x + 4$
∴ $12x = 16$
∴ $x = \frac{16}{12}$
∴ $x = \frac{8}{6}$
∴ $x = \frac{1}{6} \times 8$
∴ $x = \frac{1}{6}AB$ [∵ AB=8]]

Exercise 10.7

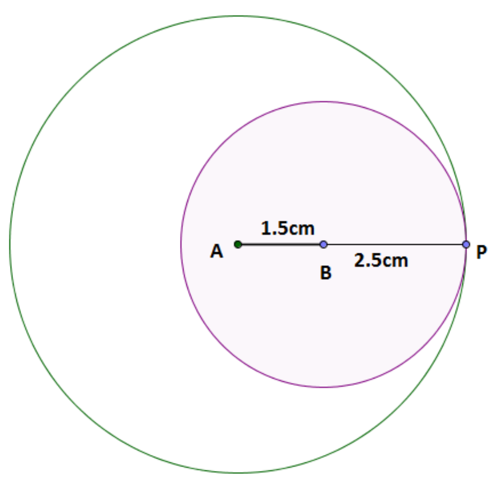
1. Draw two circles of radii 5 cm and 2 cm touching externally



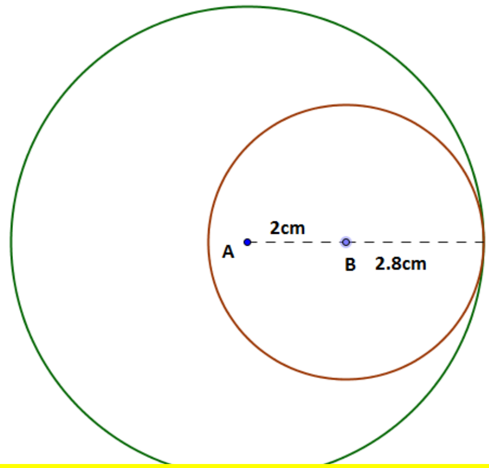
2.Construct two circles of radii 4.5 cm and 2.5 cm whose centres are at 7 cm apart.



3.Draw two circles of radii 4 cm and 2.5 cm touching internally. Measure and verify the distance between their centres.



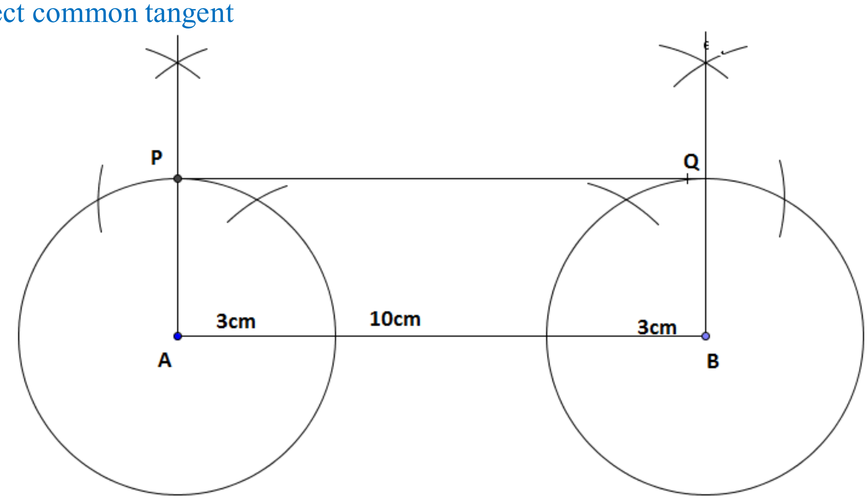
4. Distance between the centres of two circles touching internally is 2 cm. If the radius of one of the cirles is 4.8 cm, find the radius of the other circle and hence draw the touching circles.



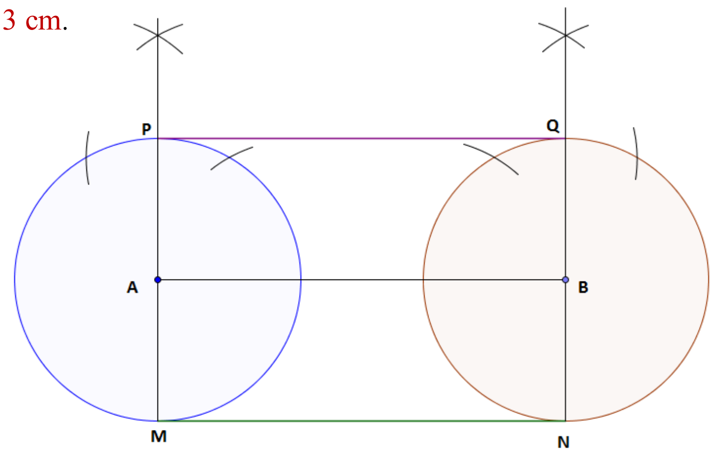
Exercise10.8

I.(A).1. Draw two congruent circles of radii 3 cm, having their centres 10 cm apart. Draw a direct common tangent

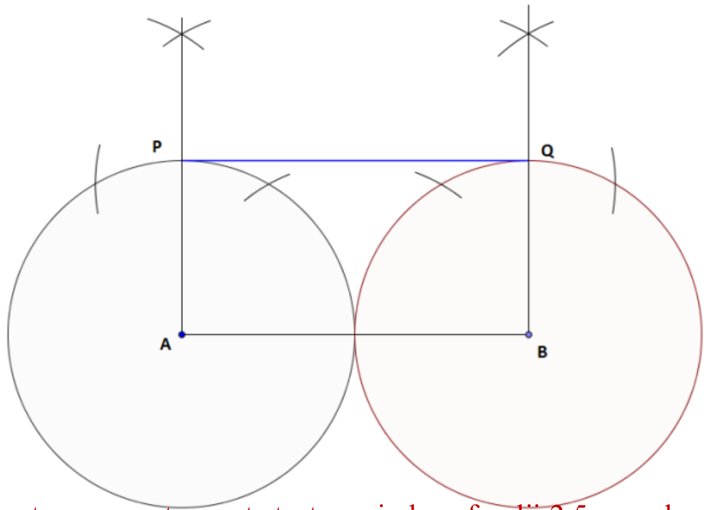
.



2. Draw two direct common tangents to two congruent circles of radii 3.5 and whose distance between them is 3 cm.

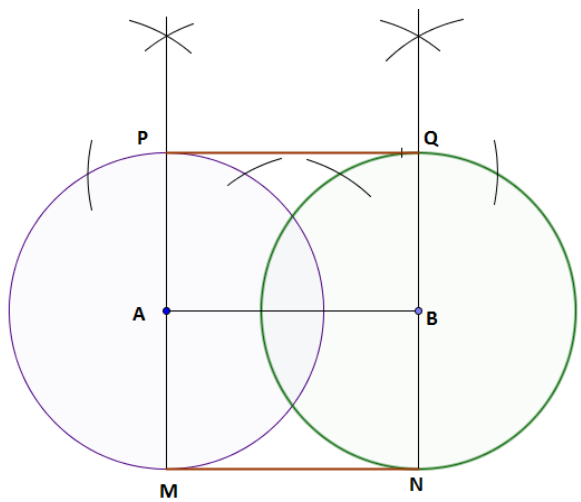


3. Construct a direct common tangent to two externally touching circles of radii.

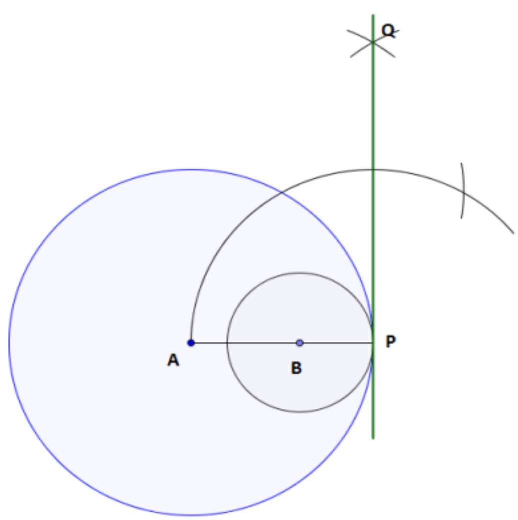


4. Draw a pair of direct common tangents to two circles of radii 2.5 cm whose centres are at 4 cm apart

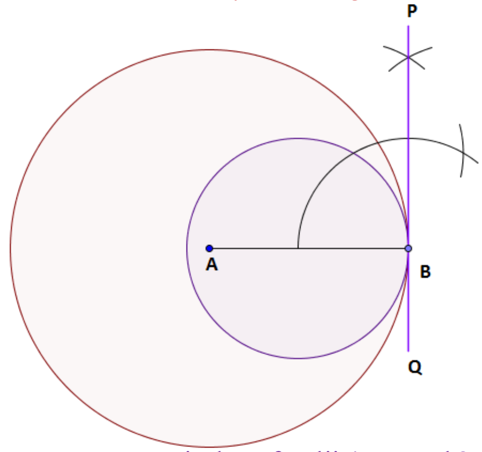
.



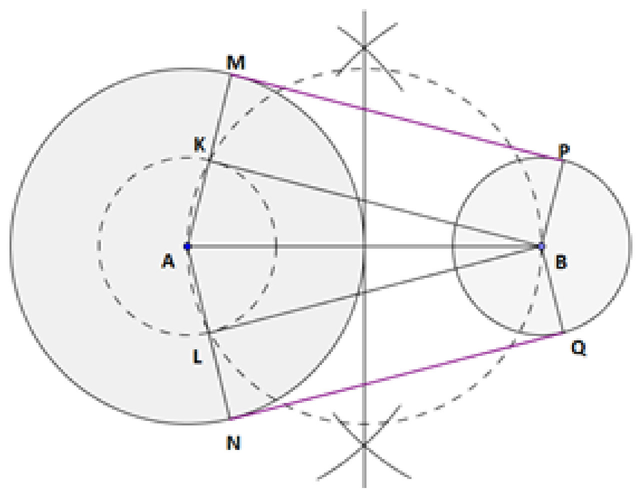
(B). 1. Construct a direct common tangent to two circles of radii 5 cm and 2cm whose centres are 3 cm apart.



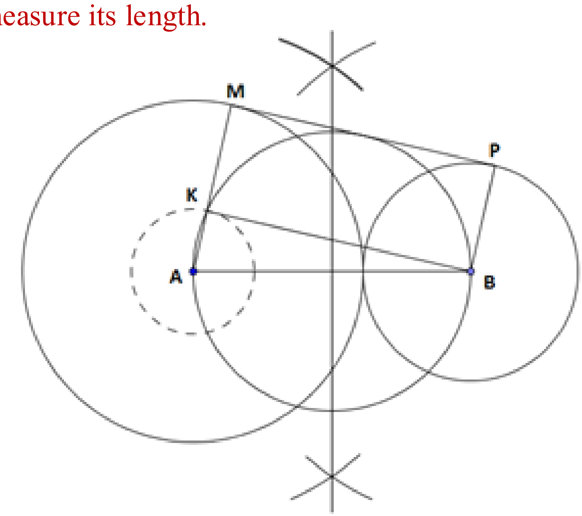
2. Draw a direct common tangent to two internally touching circles of radii 4.5 cm and 2.5 cm.



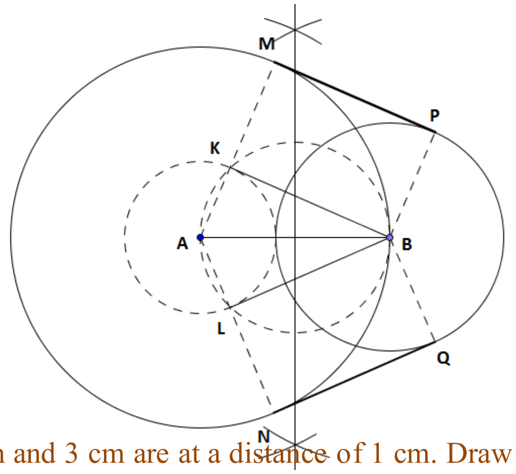
3. Construct a direct common tangent to two circles of radii 4 cm and 2 cm whose centres are 8 cm apart. Measure and verify the length of the tangent



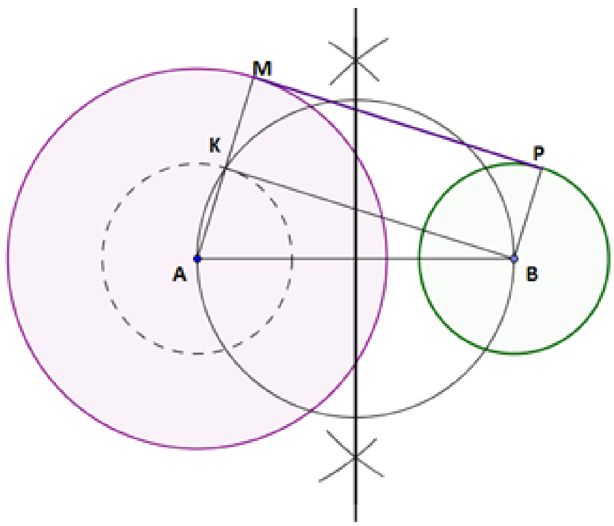
4. Two circles of radii 5.5 cm and 3.5 cm touch each other externally. Draw a direct common tangent and measure its length.



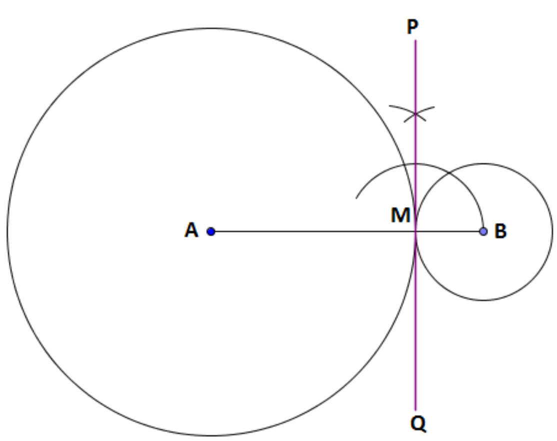
5. Draw direct common tangents to two circles of radii 5 cm and 3 cm having their centres 5 cm apart.



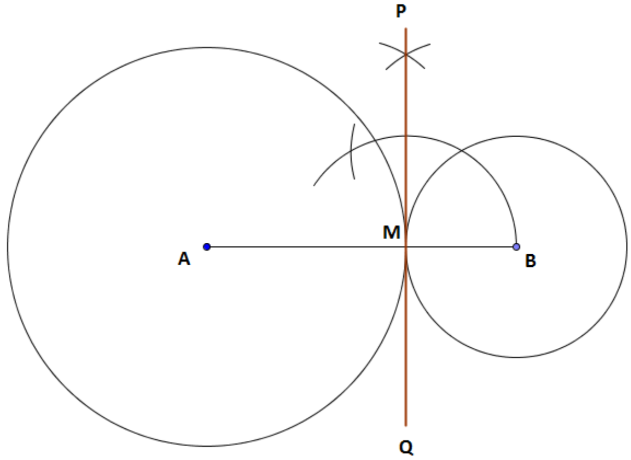
6. Two circles of radii 6 cm and 3 cm are at a distance of 1 cm. Draw a direct common tangent, measure and verify its length.



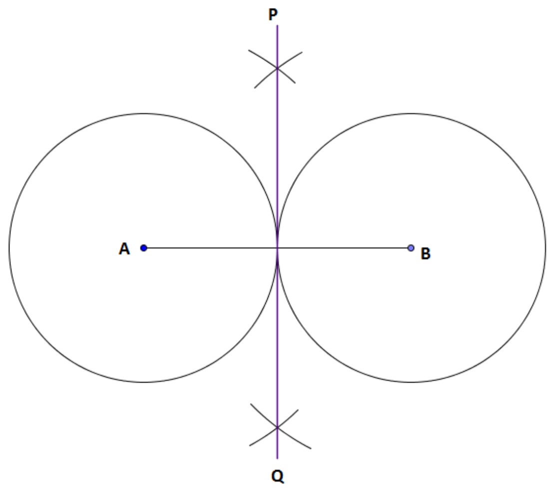
II.(A).1. Draw a transverse common tangent to two circles of radii 6 cm and 2 cm whose centres are 8 cm apart



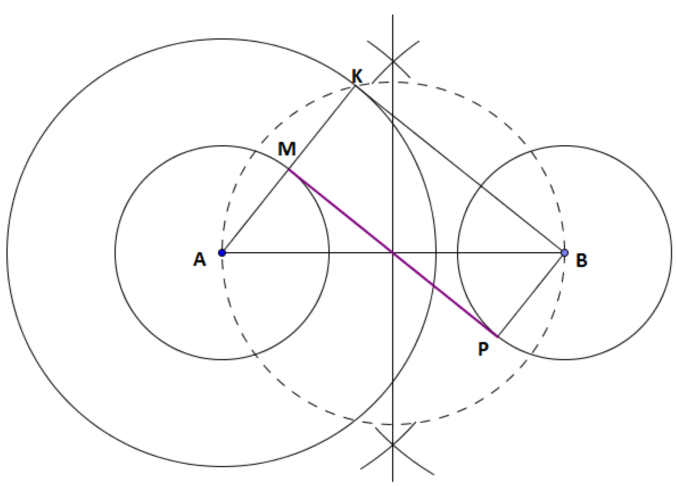
2. Two circles of radii 4.5 cm and 2.5 cm touch each other externally. Draw a transverse common tangent.



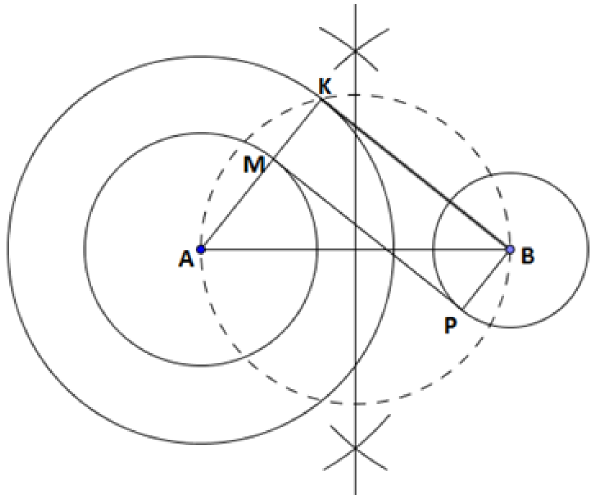
3. Two circles of radii 3 cm each have their centres 6cm apart. Draw a transverse common tangent.



(B). 1. Construct a direct common tangent to two circles of radii 5 cm and 2cm whose centres are 3 cm apart.

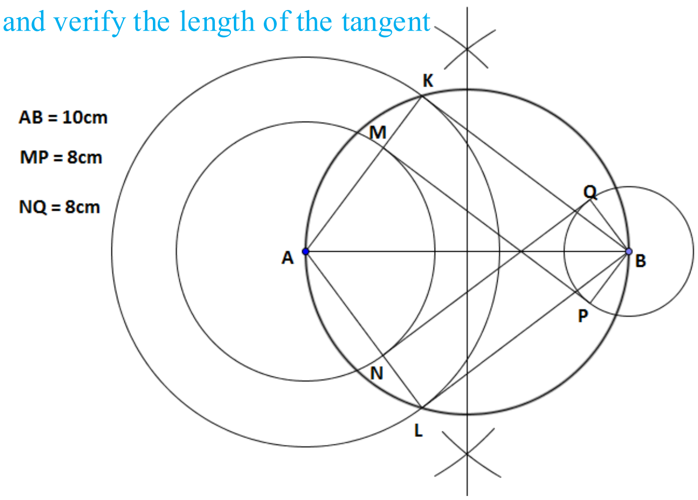


2. Draw a direct common tangent to two internally touching circles of radii 4.5 cm and 2.5 cm

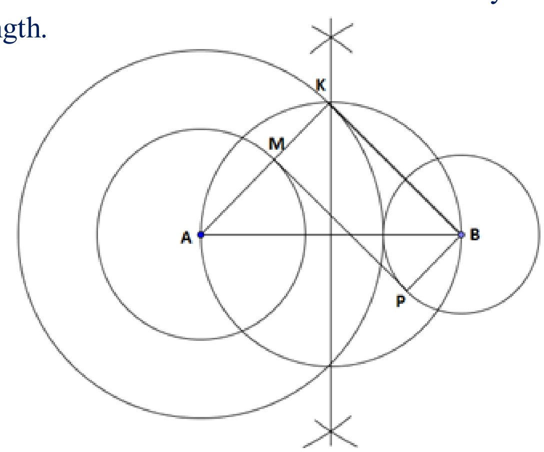


Length of the tangent = 6.2cm

3. Construct a direct common tangent to two circles of radii 4 cm and 2 cm whose centres are 8 cm apart. Measure and verify the length of the tangent

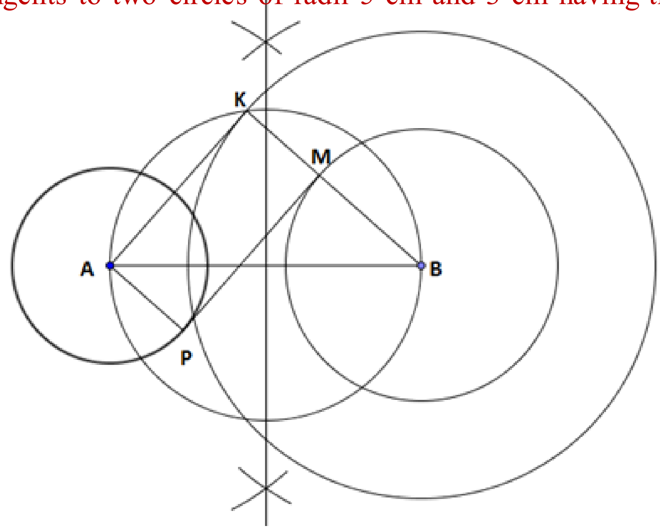


4. Two circles of radii 5.5 cm and 3.5 cm touch each other externally. Draw a direct common tangent and measure its length.



$t = \sqrt{d^2 - (R+r)^2} \Rightarrow t = \sqrt{10^2 - (4+3)^2} = \sqrt{100 - 49} = \sqrt{51} = 7.1cm$

5. Draw direct common tangents to two circles of radii 5 cm and 3 cm having their centres 5 cm apart.



$t = \sqrt{d^2 - (R+r)^2}$

$t = \sqrt{8^2 - (2.5+3.5)^2}$

$t = \sqrt{64 - 36}$

$t = \sqrt{28}$

$t = 5.29cm$

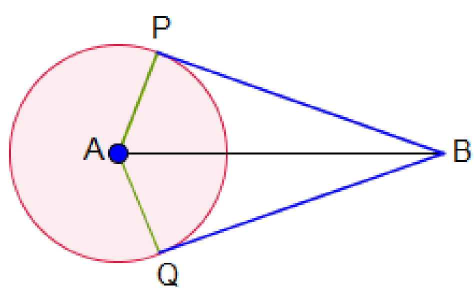
Theorem on Circles

Theorem: The tangents drawn from an external point to a circle,

(a) are equal

(b) subtend equal angles at the centre

(c) are equally inclined to the line joining the centre and the external point.



Data: A is the centre of the circle. B is an external point BP and BQ are the tangents AP , AQ and AB are joined

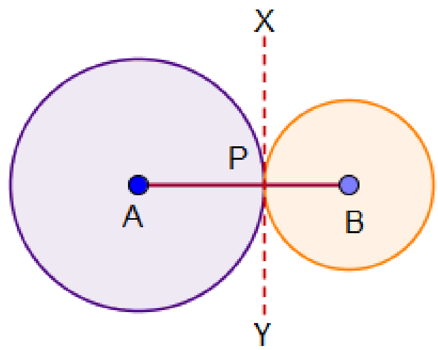
To Prove :

(a) BP = BQ
(b)∠PAB = ∠QAB
(c)∠PBA = ∠QBA

Proof:

In ΔAPB and ΔAQB,	
AP = AQ	radii of the same circle
∠APB =∠AQB =90^0	Radius drawn at the point of contact is perpendicular to the tangent
AB = AB	Common side
ΔAPB ≡ ΔAQB	RHS Theorem
(a) BP = BQ	CPCT
(b) ∠PAB =∠QAB	
(c) ∠PBA = ∠QB	

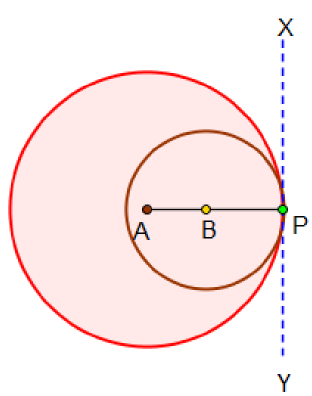
Theoem: If two circles touch each other externally, the centres and the point of contact are collinear.



Data: A and B are the centres of touching circles.
To prove : A, P and B are collinear.
Construction: Draw the tangent XPY.
Proof: In the figure,

$\angle APX = 90^0$…..(1)	Radius drawn at the point of contact is perpendicular to the tangent
$\angle BPX = 90^0$......(2)	
$\angle APX + \angle BPX = 90^0 + 90^0$	(1) + (2)
$\angle APB = 180^0$	APB is a straight angle
∴ APB is a straight line	
∴ A, P and B are collinear	

Theoem: If two circles touch each other internally, the centres and the point of contact are collinear.



Data: A and B are the centres of touching circles.
To prove : A, P and B are collinear.
Construction: Draw the tangent XPY

Proof: In the figure,

∠APX = 90^0…..(1)	Radius drawn at the point of contact is perpendicular to the tangent
∠BPX = 90^0……(2)	
∠APX=∠BPX=90^0	(1) + (2)
AP and BP are ona same straight line	
∴ APB straight line	
∴ A, P and B are collinear	